



Vector Problems

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- A boat sails 12 km [E30°N] and then 10 km more in an Easterly direction. Calculate the boats' final displacement and the total distance it sailed.
- A plane accelerates from rest at 10 m/s² [N]. A flight attendant is walking inside the plane at 1.0 m/s [S].
 - What is the attendant's velocity relative to the plane after 2.0 s?
 - What is the attendant's velocity relative to the ground after 2.0 s?
- A weather balloon is flying East at 25 km/h. It encounters a wind blowing at 20 km/h [N]. Determine the balloon's ground velocity.
- A ship travels 30 km/h [N30°E] for 10 h, then it travels at 20 km/h [W20°S] for the next 5 h. Calculate its total displacement.
- A small plane from Toronto is scheduled to fly East to Montreal at 100 km/h. It encounters a wind at 20 km/h with bearings [W30°S].
 - What will its ground velocity be if the plane's pilot allows it to drift in the wind?
 - Where [in what direction] should the pilot aim the plane so that it lands in Montreal?

Solutions:

- Use Graphical Methods

	<p>Scale: 1 cm = 2 km</p> <p>Given:</p> <p>$d_1 = 12 \text{ km [E30°N]} = 6.0 \text{ cm [E30°N]}$</p> <p>$d_2 = 10 \text{ km [E]} = 5.0 \text{ cm [E]}$</p>
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Find: d_T

- Measure d_T with a ruler and scale it back in real time measurements using the scale.
- Measure the angle θ with a protractor and report the direction of the vector d_T

$$d_T = 11.6 \text{ cm} = 23.2 \text{ km} \quad \text{and} \quad \theta = 16^\circ$$

$$\therefore d_T = 23 \text{ km [E } 16^\circ \text{ N]}$$

- Let a_p be the acceleration of the plane, and v_{att} be the velocity of the attendant.

Given: $a_p = 10 \text{ m/s}^2$ [N] and $v_{att} = 1.0 \text{ m/s}$ [S] also $\Delta t = 2.0 \text{ s}$

Find: the velocity of the plane with respect to the ground v_p

$$\begin{aligned} v_p &= (a_p)(\Delta t) \\ &= (10 \text{ m/s}^2)(2.0 \text{ s}) \\ &= 20 \text{ m/s [N]} \end{aligned}$$

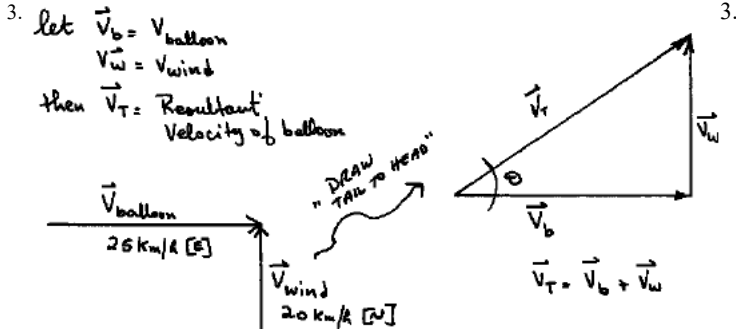
a) v_{att} relative to the plane: As far as the attendant is concerned the plane is perfectly still. Therefore $v_{p/attendant} = 0 \text{ m/s} \therefore v_{att} = 1.0 \text{ m/s [S]}$.

b) v_{att} relative to the ground: Consider yourself standing on the ground you would see the plane moving in one direction and the attendant inside the plane moving in the opposite direction. Therefore the velocity of the attendant with respect to the ground is the vector sum of the two velocities.

$$\begin{aligned} \therefore v_{ground/att} &= v_{att} + v_p \\ &= 20 \text{ m/s [N]} + 1.0 \text{ m/s [S]} \end{aligned}$$

$$= 20 \text{ m/s [N]} - 1.0 \text{ m/s [N]}$$

$$= 19 \text{ m/s [N]}$$



Solution:

a) Magnitude of Resultant V_T

b) The value of angle θ

c) State the resultant velocity of the balloon

$$V_T^2 = V_b^2 + V_w^2$$

$$V_T = \sqrt{V_b^2 + V_w^2}$$

$$= \sqrt{25^2 + 20^2}$$

$$= 32 \text{ km/h}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{V_w}{V_b}$$

$$= \frac{20}{25}$$

$$= 0.80$$

$$\therefore \theta = \text{inv tan}(0.80)$$

$$= 39^\circ$$

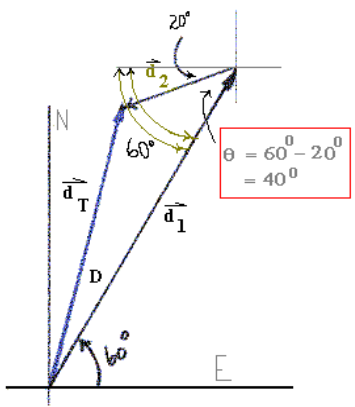
$$v_T = 32 \text{ km/h [E}39^\circ\text{N]}$$

4. Let $V_1 = 30 \text{ km/h [N}30^\circ\text{E]}$, $V_2 = 20 \text{ km/h [W}20^\circ\text{S]}$, $\Delta t_1 = 10.0 \text{ h}$, $\Delta t_2 = 5.0 \text{ h}$

Find: the first displacement d_1 and the second displacement d_2

$d_1 = V_1 \times \Delta t_1$ $= 30 \text{ km/h [N}30^\circ\text{E]} \times 10.0 \text{ h}$ $= 300 \text{ km [N}30^\circ\text{E]}$	$d_2 = V_2 \times \Delta t_2$ $= 20 \text{ km/h [W}20^\circ\text{S]} \times 5.0 \text{ h}$ $= 100 \text{ km [W}20^\circ\text{S]}$
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Sketch:



1. First you must realize that the included angle θ is 40° because d_1 spans 60° (opposite angle or "Z" Theorem).

2. Now use the Cosine Law to solve for d_T

$$d_T = \sqrt{d_1^2 + d_2^2 - 2d_1d_2 \cos 40^\circ}$$

$$= \sqrt{300^2 + 100^2 - (2)(300)(100) \cos 40^\circ}$$

$$= 232 \text{ km}$$

3. Now calculate the value of angle D using the Sine Law

$$\sin D = \frac{\sin 40^\circ}{d_T} d_2$$

$$\sin D = \frac{0.643}{232} 100$$

$$\sin D = 0.277$$

$$D = \text{inv sin}(0.277)$$

$$D = 16^\circ$$

Finally compute the angle that defines the vector $d_T = 60^\circ + 16^\circ = 76^\circ$

Therefore the final displacement of the boat is:
 $d_T = 232 \text{ km [E}76^\circ\text{N]}$

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